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SOME RESULTS FOR A QUEUE WITH BATCH ARRIVAL AND BATCH SERVICE

Dr. Hukam Singh

Department of Mathematics

M.A.J. Govt. (P.G.) College, Deeg (Bharatpur)

Rajasthan, India

Abstract

In this paper a finite server queue has been studied by considering an arrival of customers in a batch of random size, and the service being done in batches too. A generating function for the time dependent queue length and the limiting distribution of service have been obtained. To make the use of the results convenient an expression for stochastic mean is also derived.

FORMULATION OF THE PROBLEM

To formulate a queueing system we consider such system in which arrivals occur in batches in poisson fashion and the batches are of random size, i.e. $M^X/M^X/C$. Customers are provided service immediately but in batches by C service channels whose service time is distributed according to a negative exponential law. Identical statistical properties are hold by all service channels.

In particular let, the probability of the arrival of a batch in a short time interval

$$\delta t = \lambda \delta t + 0\ (\delta t).$$

The probability of a particular service ending is

$$\delta t = \mu \delta t + 0\ (\delta t).$$

If there are n customers in the system (all receiving service) then, the probability of a departure is

$$\delta t = n\,\mu \;\; \delta t + 0\,(\delta t).$$

The probability that an arriving batch contains r customers = π_r (r = 1, 2, ……..) independent of all previous batch sizes.

<u>We define</u>

$$\text{II}\,(x) = \sum_{r=1}^{\infty} \pi_r \;\; x^r$$

$$X_r = \sum_{s=r}^{\infty} \pi_{s+1} \quad , \quad \text{for } r \geq 0$$

$$X\,(x) = \sum_{r=0}^{\infty} X_r \; . \, x^r$$

$$= \frac{1 - \text{II}\;(x)}{(1 - x)} \qquad \text{…..(1)}$$

so that II (x) is the p.g.f. of batch size and X_r the probability that the batch size is greater than r.

<u>PROBABILITY ANALYSIS</u> (A Generating Function)

let $P_{ij}\,(t) = \text{Prob.}\,[N\,(t) = j \;/\; N\,(O) = i]$

.....(2)

Where N(t) denotes the number of customers in the system at time t. We get the following equation in usual way.

$$P_{ij}(t+\delta t) = \lambda \delta t \sum_{r=1}^{j} \pi_r P_{ij-r}(t) + [1-(\lambda + j\mu)\delta t] P_{ij}(t)$$

$$+ C\mu\,\delta t \sum_{S=1}^{c-1} P_{ij+s}(t) + 0(\delta t) \qquad .....(3)$$

Dividing through out by δt and letting $\delta t \to 0$, we obtain the following defferential difference equations-

$$\frac{d}{dt}[P_{ij}(t)] = \lambda \sum_{r=1}^{j} \pi_r P_{ij-r}(t) - (\lambda + j\mu) P_{ij}(t)$$

$$+ C\mu \sum_{s=1}^{c-1} P_{i,\,j+s}(t) \qquad .....(4)$$

solving equation (4) by introducing the p.g.f.

$$\theta_i(x,t) = \sum_{j=0}^{\infty} P_{i,j}(t) \,.\, x^j$$

with the initial condition

$$\theta_i(x,o) = x^j \qquad .....(5)$$

Multiplying equation (4) by x^j and summing over j,

we obtain –

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$$\sum_{j=0}^{\infty} \frac{d}{dt} P_{i,j}(t) \,.\, x^j = \lambda \sum_{r=1}^{j} \pi_r \sum_{j=0}^{\infty} P_{i,j-r}(t) \,.\, x^j$$

$$-\sum_{j=0}^{\infty} (\lambda + j\mu) P_{i,j}(t) \,.\, x^j$$

$$+\sum_{j=0}^{\infty} C\,\mu \sum_{S=1}^{c-1} P_{i,j+s}(t) \,.\, x^j$$

or

$$\frac{\partial \theta_i}{\partial t} = \lambda \sum_{r=1}^{\infty} \pi_r \,.\, x^r \sum_{j-r=0}^{\infty} P_{i,j-r}(t) \,.\, x^{j-r}$$

$$-\lambda \sum_{j=0}^{\infty} P_{i,j}(t) \,.\, x^j - \mu \sum_{j=0}^{\infty} j\, P_{i,j}(t) \,.\, x^j$$

$$+\sum_{j=0}^{\infty} \sum_{S=j}^{c-1} C\,\mu\, P_{i,j+s}(t) \,.\, x^{j+s} \,.\, x^{-s}$$

or

$$\frac{\partial \theta_i}{\partial t} = \lambda.\, \Pi(x) \,.\, \theta_i - \lambda\, \theta_i - \mu\, x \frac{\partial \theta_i}{\partial x} + \frac{C\mu x}{x-1} \theta_i$$

$$= [\lambda.\, \Pi(x) - \lambda + \frac{C\mu x}{x-1}]\, \theta_i - \mu\, x \frac{\partial \theta_i}{\partial x}$$

or

$$\frac{\partial \theta_1}{\partial t} + \mu\, x \frac{\partial \theta_i}{\partial x} = [\lambda \,.\, \Pi(x) - \lambda + \frac{C\mu x}{x-1}]\, \theta_i \qquad \text{.....(6)}$$

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This is a special case of Lagrange's linear equation and can be solved with the help of following subsidiary equation (Piaggio[1] for a general account or Bailey[2] for some applications of generalized birth-death models).

$$\frac{dt}{1} = \frac{dx}{\mu x} = \frac{d\theta_i}{[\lambda \;\; \Pi(x) - \lambda + \frac{C\mu x}{x-1}]\,\theta_i} \qquad \text{.....(7)}$$

From first two expressions, we have

$$\frac{dt}{1} = \frac{-dx}{\mu x}$$

or $$\mu \, dt = \frac{1}{x} dx$$

or $$\mu \, dt - \frac{1}{x} dx = 0$$

or $$\int \mu dt - \int \frac{1}{x} dx = \int 0$$

or $$\mu t - \log x = \log C$$

or $$\log e^{-\mu t} + \log x = \log C$$

or $$e^{-\mu t} . x = \text{Constant}$$

or $$x \; . \; e^{-\mu t} = \text{Constant.} \qquad \text{.....(8)}$$

From last two expressions, we have

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$$\frac{dx}{\mu x} = \frac{d\theta_i}{[\lambda\, \Pi\ (x)\ - \lambda + \frac{C\mu x}{x-1}]\ \theta_i}$$

or $$\frac{[\lambda\,(\Pi\ (x)\ -1) + \frac{C\mu x}{x-1}]}{\mu x}\,dx = \frac{d\theta_i}{\theta_i}$$

or $$\int \frac{\lambda\,(\Pi\,(x) - 1)}{\mu x}\,dx + \int \frac{C}{x-1}\,dx - \int \frac{d\theta_i}{\theta_i} = \int 0$$

or $$\int \frac{\lambda\,(\Pi\,(x) - 1)}{\mu x}\,dx + C\log(x-1) - \log\theta_i = \log C$$

or $$\log\, e^{-\int \frac{\lambda\,(\Pi\ (x)-1)}{\mu x}dx} + \log(x-1)^{-C} + \log\theta_i = \log C$$

or $$e^{-\int \frac{\lambda\,(\Pi\ (x)-1)}{\mu x}dx}\,.\,(x-1)^{-C}\,.\,\theta_i = \text{Constant.} \qquad \text{.....(9)}$$

Hence the general solution can be given as,

$$\theta_i\ (x,t) = (x-1)^{-C}\,.\,e^{-\int \frac{\lambda(\Pi\,(x)-1)}{\mu x}dx}\,.\,F_1\,[e^{-\mu t}\,.\,x] \qquad \text{.....(10)}$$

where F_1 denotes an arbitrary function.

with the help of (5) we can obtain N (j) and hence θ_i (x, t).

Let us consider the limiting form of

$\theta_i\ (x,t)$ as $t \to \infty$, we have

$$q(x) = \lim_{t\to\infty} \theta_i \,(x, t)$$

or

$$q(x) = (x-1)^{-C} \,.\, e^{-\int \frac{\lambda(\Pi(x)-1)}{\mu x} dx} \qquad \text{.....(11)}$$

Equation (11) confirms the existence of an equilibrium distribution independent of the initial state of the queue.

STOCHASTIC MEAN

From the solution of θ_i (x, t), we can obtain an expression for the stochastic mean as given below.

$$E\,[N\,(t)\,/\,N\,(0) \;\; = i\,] = \left[\frac{\partial \theta_i \,(x, t)}{\partial\, x}\right]_{x=1} \qquad \text{.....(12)}$$

CONCLUSION

The applications of this paper are in the Banks, Telephone Exchanges, Hospital admission system, Seaports, Toll booths, Court of Law wherein servers are Judges and customers are cases and information transmission systems wherein messages containing a small random number of characters (a batch of characters) arrive according to a poisson process and must be transmitted to some destinations.

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