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OPTIMAL PREDICTION OF QUEUE LENGTHS IN THE STATIONARY SINGLE SERVER QUEUE WITH RENEGING

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ABSTRACT

We present optimal mean square predictors for queue Lengths in the stationary single server queues with reneging which is based on a queue length measurement. The conditional expectation predictors are observed for a number in the queue with a previous number in the queue as the basis for the prediction.

INTRODUCTION

In this paper we describe optimal mean square predictors for queue lengths in the stationary single server queues with reneging. The analysis extends the previous work, and gives a queue length predictor which is quite different, in that it drops some terms that are included in the paper of Pagurek et al. (1988). Many authors have interesting studies on prediction of queues specially for single server queueing systems. Stanford et al. (1983) presented the optimal prediction of times and queue lengths in the GI/M/1 queue. Pagurek et al. (1988) has also studied the optimal prediction of times and queue lengths in the M/G/1 queue. The fundamental difference between GI/M/1 and M/G/1 queues are that the imbedded Markov Chain is the sequence of queue lengths observed at departure instants rather than arrival instants and the queue length left behind by a departing customer is also the number of arrival during its system time. Woodside et al. (1984) analysed the optimal prediction of queue lengths and delays in GI/M/m multiserver queues.

We propose the optimal mean square prediction for future queue lengths in the stationary single server queues with reneging for several variables which is based on a queue length measurement. Specially, the conditional expectation predictors have been observed for a number in the queue with a previous number in the queue as the basis for the prediction. Such types of problems can be seen in the managing queueing system such as multiprocessing computers and data communications networks, adaptive routing and in overload of switching computers and industrial job shop systems.

FORMULATION OF THE PROBLEM AND MATHEMATICAL MODELLING:

This work is concerned with the stationary M/G/1 single server queues with reneging with arrival rate λ and service rate μ for each server. The mean arrival rate where there are n customers in the system is λ_n and service time distribution function for a customer is $F_x(t)$, traffic intensity $\rho = \frac{\lambda}{n^{\alpha}\mu}$, the n^{th} customer (denoted by C_n) leaves behind him D_n customers when he departs from the queue and has service time X_n. The expectation and variance of any random variable y will be donted by $\overline{Y} = E\{y\}$ and $\alpha_y^2 = var\{y\}$.

Now the well known one step transition probabilities P_{ij} (P_{ij} (n) for n steps) and steady-state probabilities π_i are given below: (Gross and Harris 1974)

$$P_{ij} = P_r\{D_{n+1} = j \mid D_n = i\} = \begin{cases} K_j, & i = 0 \\ K_{j-i+1}, & i \geq 1,\ j \geq i-1 \\ 0, & elsewhere \end{cases}$$

where

$$K_l = \int_0^{\infty} \frac{e^{-\lambda t}(\lambda t)^l}{l!}\, dF_x(t), \qquad l \geq 0 \text{ and}$$

$$\pi_i = P_r\{D_n = i\}, \qquad i \geq 0$$

The stationary distribution for the chain $\{D_n\}$, n = 0, 1, 2. In this case Chapman kolmogorov equation becomes

$$P_{ij}^{\,n} = \sum_{l=i-1}^{\infty} P_{il}\, P_{lj}^{n-1} \qquad \text{.... (1)}$$

for n step transition probabilities

$$P_{ij}^{n} = P_r\,\{D_{k+n} = j \mid D_k = i\}$$

Finally let θ be distributed as the number of arrivals that occur during an arbitrary service time. Then

$$P_r\,\{\theta = 1\} = K_l\,, l > 0$$

and also

$$\bar{\theta} = \frac{\lambda}{n^{\alpha}\mu}, \alpha_{\theta}^{2} = \frac{\lambda}{n^{\alpha}\mu} + \lambda^{2}\alpha_{x}^{2} \qquad \text{.... (2)}$$

QUEUE LENGTH PREDICTION

Fundamental Results:

The predictor is $E\,\{D_n \mid D_0 = i\}$ which is well known to minimize the mean square prediction error and its conditional variance and overall mean squared error (MSE) are given below:

$$\left.\begin{aligned} & var\,\{D_n|D_0=i\}=E\,\{D_n^2|D_0=i\}-E\,\{D_n|D_0=i\}^2 \\ & and \\ & MSE\,(D_n)=E\,\{(D_n-E\{D_n|D_O\})^2\}=E\,\{var\;\{D_n\mid D_0\}\} \\ & \qquad\qquad =\sum_{i=0}^{\infty}\pi_i\,var\,\{D_n\mid D_O=i\} \end{aligned}\right] \quad \ldots.(3)$$

(Papoulis (1986) and Sage and Melsa (1971))

By applying Chapman-Kolmogorov equation (1), we can find the recursive results of the desired quantities:

$$E\{D_n\mid D_O=i\}=\sum_{j=0}^{\infty} jP_{ij}^n=\sum_{l=max\,(O,i-1)}^{\infty}P_{i1}\,E\,\{D_{n-1}\mid D_O=l\} \quad \ldots.(4)$$

and

$$E\{D_n^2\mid D_O=i\}=\sum_{j=0}^{\infty} j^2P_{ij}^n=\sum_{l=max\,(O,i-1)}^{\infty}P_{il}\,E\,\{D_{n-1}^2\mid D_O=l\} \quad \ldots.(5)$$

from (4) & (5) we can obtain the conditional variance var $\{D_n \mid D_O=i\}$ and MSE $\{D_n\}$ with the help of (3). In case of i = 0 there is no need to calculate these quantities explicitly as

$$E\,\{D_n^k\mid D_O=0\}=E\,\{D_n^k\mid D_O=1\},\ \text{for } k=1,2$$

(therefore $P_{Oj}=P_{1j}\ \forall j$). Similarly

$$var\,\{D_n\mid D_O=0\}=var\,\{D_n\mid D_O=1\}$$

EFFICIENT COMPUTATION OF THE QUEUE-LENGTH PREDICTIONS:

Now we will replace the infinite sums by finite sums of (3), (4) & (5). In the first case $i \geq n$, the expected queue size left behind is given by

$C_n = i - n$ + (the expected number that arrive during the n service times)

since the number of arrivals during the service times are independent,

$$E\{D_n \mid D_O = i\} = i - n + n\overline{\theta} = i - n + n^{1-\alpha}\frac{\lambda}{\mu}, i \geq n \qquad \ldots.(6)$$

Similarly

$$var\{D_n \mid D_O = i\} = n\,\alpha_\theta^2$$

$$= n\left[\frac{\lambda}{n^\alpha \mu} + \lambda^2\,\alpha_x^2\right], \quad i \geq n \qquad \ldots.(7)$$

for the second case $i < n$ the summation can be simplified easily. For $l \geq n - 1$, we have

$$E\{D_{n-1} \mid D_O = l\} = (l - n + 1) + (n - 1)\frac{\lambda}{n^\alpha \mu} \qquad \ldots.(8)$$

hence

$$E\{D_n \mid D_O = i\} = \sum_{l=i-1}^{n-2} P_{il}\, E\{D_{n-1} \mid D_O = l\} \sum_{l=n-1}^{\infty} P_{il}\left[l - (n-1)\left(1 - \frac{\lambda}{n^\alpha \mu}\right), \ 0 < i < n\right] \ldots.(9)$$

Thus

$$E\{D_n \mid D_O = i\} = (i - n) + n^{1-\alpha}\frac{\lambda}{\mu} + \sum_{l=i-1}^{n-2} P_{il}\left[E\{D_{n-1} \mid D_O = l\}\right.$$

$$-l+(n-1)\left(1-\frac{\lambda}{n^{\alpha}\mu}\right)\Bigg], \quad 0 \le i < n \qquad \ldots (10)$$

By the same procedure we can replace (5) for $i < n$ therefore,

$$E\left\{D_n^2 \mid D_O = i\right\} = \sum_{l=i-1}^{n-2} P_{il}\, E\left\{D_{n-1}^2 \mid D_O = l\right\} - \left\{(l-n+1)(n-1)\frac{\lambda}{n^{\alpha}\mu}\right\}^2$$

$$+n\left(\frac{\lambda}{n^{\alpha}\mu}+\lambda^2\alpha_x^2\right)+\left(i-n-n^{1-\alpha}\ \frac{\lambda}{\mu}\right)^2,$$

$$0 < i < n \qquad \ldots. (11)$$

Result (10) and (11) are also valid for $i \ge n$ since the summation disappears.

Now substituting (7) in (3), the mean squared error becomes

$$MSE\left\{D_n\right\} = \sum_{i=0}^{n-1} \pi_i\ var\left\{D_n \mid D_O = i\right\} + \sum_{i=n}^{\infty} \pi_i\ n\left(\frac{\lambda}{n^{\alpha}\mu}+\lambda^2\alpha_x^2\right)$$

$$= \sum_{i=0}^{n-1} \pi_i\ var\left\{D_n \mid D_O = i\right\} + \ n\left(\frac{\lambda}{n^{\alpha}\mu}+\lambda^2\alpha_x^2\right)$$

$$\left(1-\sum_{i=0}^{n-1}\pi_i\right) \qquad \ldots. (12)$$

CONCLUSIONS:

Computable results have been obtained for the queue size which is important in case of reneging. The departure-instant approach to prediction followed here is more accurate in the sence that errors are smaller than in the arrival-instant.

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