

ACCRETION ON TO MAGNETISED NEUTRON STARS

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Abstract

We have investigated boundary layer accretion and resolved that it is not applicable for the case of white dwarf and neutron star having magnetic field of an order. 10^7 G and 10^{12} G respectively. We have presented the picture of neutron star having magnetic field which disrupts the accretion flow and also which is a quasispherical far from the neutron star.

Key words: neutron stars; magnetic field; radio pulsars; magnetars.

Introduction

Accretion of matter with significant angular momentum on to neutron star is accompanied by the formation of disks of accreting material. Disks should arise around neutron star, white dwarfs, black holes and super massive black holes. The theory of the structure and radiation of stationary disks was constructed by Lynder Bell (1969), Shakura (1973), Shakura and Sunyaev (1973) and Pringle and Rees (1973) and is determined by three parameters : the mass of the neutron star M , the accretion rate. \dot{M} (or the total disk luminosity $L = \eta \dot{M} c^2$, where η is the efficiency of energy generation) and the parameter, Characterizing the level of turbulence and/or chaotic small scale magnetic field:

$$\alpha = \frac{v_t l_t}{v_s H} + \frac{B^2}{4\pi\rho v_s^2} \dots\dots\dots (1)$$

where v_t and v_s are the turbulent and thermal velocities of the matter respectively, $\frac{B^2}{2\pi}$ be the energy density of the chaotic magnetic field, $\frac{\rho v_s^2}{2}$ is the thermal energy density of the matter in the disk. It is the turbulent mixing length and H be the half thickness of the disk.

Rees, Pacholczyk and Pringle (1973) and subsequently Lightman and Eardley (1974), raised the question of the stability of disk accretion. Lightman and Eardley (1974) and also Lightman (1974, a, b) presented the stability of a dynamic equation for disk accretion and came to the conclusion that stationary accretion is unstable in zone A and stable in zones B and C. The time dependent equations of disk accretion and their analysis were presented by Shakura and Sunyaev (1975). Also Shibasaki and Hoshi (1975) demonstrated the possibility of thermal instability. It is evident that an accretion disk provides an efficient machine for the extraction of the energy. We know the existence of an inner boundary layer in the case of where the disk extends down to the surface of the accreting star. For neutron stars, runaway nuclear burning may occur at the surface and give rise to the X-ray bursters. It is observed that in many systems the accreting star possesses a magnetic field strong enough to disrupt the inner regions of the disk and channel the accretion flow on to the magnetic pole caps. In this situation, the accretion flow resembles more nearly free-fall on to the stellar surface. A black hole, on the other hand, can not possess an intrinsic magnetic field, so column accretion of this kind cannot occur.

Boundary Layers

For steady thin accretion disks we assume that the angular velocity $\Omega(R)$ in the disk remains very close to the Keplerian value i.e. $\Omega(R) = \left(\frac{GM}{R^3}\right)^{1/2}$ until the accreting matter enters a boundary layer of radial extent b just outside the surface $R = R_s$ of the accreting star. Within this boundary layer Ω must decrease from a value $\Omega(R_s + b) = \Omega(R_s)$ to the surface angular velocity $\Omega_s < \Omega(R_s)$. Let us assume $b \ll R_s$, we get the inner boundary condition

$$C = -M(GMR)^{1/2} \dots\dots\dots (2)$$

where the constant C , is related to the rate at which angular momentum flows into the compact star, or equivalently, the couple exerted by the star on the inner edge of the disk. In order to justify the assumption $b \ll R_s$ we now consider the Euler equation

$$p \frac{dv}{dt} + p v \cdot \nabla v = -\nabla p + f \dots\dots\dots (3)$$

where v be the velocity field, ρ the density, p be the pressure, and f the force density (the force per unit volume). The radial component of the eq (2) takes the form under thin disk approximation,

$$v_R \frac{dv_R}{dR} - \frac{v_q^2}{R} + \frac{1}{\rho} \frac{dP}{dR} + \frac{GM}{R^2} = 0 \quad \dots\dots\dots (4)$$

The boundary layer is by definition that region of the disk in which $v_q^2 < v_k^2 = \frac{GM}{R}$. Hence the gravity term $\frac{GM}{R^2}$ in eq. (4) must be balanced either by $v_R \frac{dv_R}{dR} \frac{v}{b}$ or the pressure gradient

$$P - 1 \frac{dP}{dR} \frac{c_s^2}{b} \quad \text{where we have taken } \rho = \rho c_s^2 \quad \text{and by setting } \frac{d}{dR} - b^{-1}.$$

Since $c_s^2 > v_R^2$, hence, the boundary layer given by the eq. (4) is approximately.

$$\frac{C_s^2}{b} \frac{GM}{R^2}$$

Let the typical scale-height of the disk in the x-direction is H , we get

$$H = C_s \left(\frac{R}{GM} \right)^{\frac{1}{2}} R$$

where C_s is the sound speed.

Let us assume that H and C , just inside $R + b$ are smaller to their values just outside, the eq. (5) and eq. (6) may be combined to give

$$b \frac{R^2}{GM} C_s^2 \frac{H^2}{R} \quad \dots\dots\dots (7)$$

which proves the assumption $b \ll R$, for

$$b \frac{H^2}{R} \ll H \ll R \quad \dots\dots\dots (8)$$

A picture presented by the boundary layer geometry gives radiation through a region of radial extent H on the two faces of the disk. If the accretion rate, and hence the density in this region is high enough it will be optically thick and radiate roughly as a blackbody of area:

$$2 R H \times 2 \dots\dots\dots (9)$$

So the luminosity emitted by this are must be

$$1/2 L_{acc} = \frac{GMM}{2R_*} \dots\dots\dots (10)$$

Hence boundary layer blackbody temperature T_{BL} is given by

$$4 R_* H_o T_{BL}^4 = \frac{GMM}{2R_*} \dots\dots\dots (11)$$

Where σ be the Stefan-Betzmann constant. Because of the blackbody assumption we cannot discuss the internal structure of the boundary layer. However, this is achieved at the expense of the assumption that the viscosity in the boundary layer may adjust itself suitably to make the structure self-consistent. The great complexity of the gas flow in the boundary layer has so far presented a self-consistent treatment of this idea. A recent alternative treatment shows that the optically thin boundary layer gas is likely to be thermally unstable to turbulent viscous heating.

For the characteristic disk blackbody temperature T_* , we get

$$T_* = \frac{(3GMM)}{8 R_*^3 \sigma} \dots\dots\dots (12)$$

Hence, one may easily obtain

$$T_* \frac{(R_*)^{1/4}}{3H} T_* \dots\dots\dots (13)$$

Accretion on to magnetized neutron stars

The above picture of the boundary layer accretion may only be relevant if the disk extends right down to the surface of the accreting star. But often this will not be the case, as white dwarfs and especially neutron stars having magnetic fields of an order $\leq 10^7$ G and 10^{12} G respectively which is strong enough to disrupt the disk flow. In general the interaction of the disk and magnetic field is exceedingly complex. We shall now consider the simple case in which the stellar magnetic field disrupts an accretion flow which is quasi-spherical far from the

star. Let us consider a dipole-like magnetic field, where the field strength varies roughly as

$$B \propto \frac{u}{r^3} \quad \dots\dots\dots (14)$$

where r be the radial distance from the star of radius R* here u the magnetic moment and equal to

$$u = B_* R_*^3 \quad \dots\dots\dots (15)$$

is constant for surface field strength B* at r = R*.

In the hydromagnetic approximation Boyd and Sanderson (1969) have obtained

$$j = o \frac{(E + v \times B)}{C} \quad \dots\dots\dots (16)$$

where j is current density, v the velocity of the fluid, and o the electrical conductivity and assumed constant. Hence, in view of eq. (16) one obtains the magnetic pressure

$$P_{mag} = \frac{u^2}{8 r^6} \quad \dots\dots\dots (17)$$

It is increasing steeply as the matter approaches the stellar surface. This magnetic pressure controls the matter flow and hence, disrupt the spherically symmetric in fall a radius r_M where it first exceeds the ram and gas pressures of the matter. For highly supersonic accretion, we know that the ram pressure term pv² where v v_{ff} (free-fall) = $\frac{(2GM)^{1/2}}{r}$ and we define the accretion rate as

$$|pv| = \frac{M}{4 r^2}$$

Now setting $P_{mag}(r_M) = \frac{pv^2}{r_M}$, one obtains

$$P_{mag}(r_M) = \frac{u^2}{8 r_M^6} \frac{(2GM)^{1/2} M}{4 r_M^{5/2}} \quad \dots\dots\dots (18)$$

In review of eq. (18), we get

$$r_M = 5.1 \times 10^8 M_{16}^{-2/7} M_1^{-1/7} u_{30}^{4/7} \text{ cm} \quad \dots\dots\dots (19)$$

where u_{30} is u in units of 10^{30} G cm^3 . It is to be noted that a neutron star with $B_* = 10^{12} \text{ G}$, $R_* = 10^6 \text{ cm}$, has $u = 1$ as does white dwarf for $B_* = 10 \text{ G}$, $R_* = 5 \times 10^8 \text{ cm}$. The eq. (19) suggests that observable effects are quite possible where accretion takes place on to magnetized neutron star or white dwarf. Now let us replace M in eq. (19) in terms of accretion luminosity by using following equation.

$$L_{acc} = \frac{GMM}{R_*}$$

We obtain

$$r_M = \begin{cases} 5.5 \times 10^8 M_1^{1/7} R_9^{-2/7} L_{33}^{-2/7} u_{30}^{4/7} \text{ cm} \\ 2.9 \times 10^8 M_1^{1/7} R_6^{-2/7} L_{37}^{-2/7} u_{30}^{4/7} \text{ cm} \end{cases} \quad \dots\dots\dots (20)$$

With parameterizations appropriate for white dwarf and neutron star acceptors respectively $\left(L_{33} = \frac{L_{acc}}{10^{33}} \text{ erg s}^{-1} \right)$. The quantity matter will flow along field lines. One interesting consequence of eq. (19) or (20) is connected to the model of an AM Herculis (polar) system, with a highly magnetized white dwarf, ‘phase-locked’ to the binary period. The accretion flow is controlled by the magnetic field, and no accretion disk is believed to form.

Let us return to the consideration of disk accretion where the condition for magnetic disruption at cylindrical radius $R = R_M$ must be that the torque exerted by the magnetic field on the disk at R_M should be of the order of the viscous torque. In a steady state it is equal to the transport rate of specific angular momentum $M R_M^2 \Omega \left(R_M \right)$. Several estimates of the R_M have been made to yield

$$R_M \approx 0.5 r_M \quad \dots\dots\dots (21)$$

Of course, the precise results must depend on the inclination of the dipole axis to the disk plane. In general, the accreting star and the magnetic field rotate with an angular velocity Ω about an axis perpendicular to the disk. Inside $R = R_M$ the matter must flow along the

field-lines, if steady accretion is to occur, we must have $w^* < 1$ (R_M) where $w^* = \frac{v_{\infty}}{v_{\text{esc}}(R_M)} = \left(\frac{GM}{R_M^2}\right)^{\frac{1}{2}}$. Otherwise particles attached to the field lines at $R = R_M$ would spiral outwards to larger R_1 repelled by the centrifugal barrier at R_M . So we define the ‘fastness parameter’.

$$w^* = \frac{v_{\infty}}{v_{\text{esc}}(R_M)} < 1 \quad \dots\dots\dots (22)$$

In view of eqs. (20) and (21) a magnetized neutron star will have $r_M \approx R_M 10^8$ cm for typical parameters : $M_1 \approx 1.4 M_{\odot}$, $L_{37} \approx 1$, which is well outside the stellar radius $R_* \approx 10^6$ cm. For low values of magnetic field, $B_* \leq 10^6$ G we obtain r_M and R_M may only be less than $R_* \approx 10^6$ cm. Hence, we hope accretion on to neutron star to be controlled by the magnetic field near the surface in many cases. It is to be noted that accretion is more efficient as a power source the more compact the accreting object is. Any stellar mass compact object will have a radius R_* less than R_{circ} (circularization radius) for realistic binary parameters. Here $R_M < R_{\text{circ}}$ for a neutron star, so we see that disk formation is not affected in this case. However, the question of wind accretion is much more difficult, and whole subject of stellar wind accretion by magnetized neutron star is one of considerable uncertainty. The accretion flow is channeled on to only a small fraction of the stellar surface in case of magnetically controlled accretion. Hence the area of an accreting pole cap is a fraction.

$$f_{\text{disk}} \approx \frac{R_*}{2R_M} \quad \dots\dots\dots (23)$$

In view of eqs. (20) and (21) we get

$$f_{\text{disk}} \approx 10^{-1} - 10^{-4} \quad \dots\dots\dots (24)$$

depending on L and u . By changing $R_M \rightarrow r_M$ for the case of quasi-spherical accretion, we get

$$f_{\text{sph}} \approx 0.5 f_{\text{disk}}$$

Lewis and heuvel (1983) have presented for $w^* \ll 1$ (for a neutron star) as,

$$\frac{\dot{M}_{\text{pulsar}}}{\dot{M}_{\text{pulsar}}} = 1.5 \times 10^{-5} \left(\frac{P_{\text{pulsar}}}{10^3}\right) L_{37}^{6/7} \nu^{-1} \quad \dots\dots\dots (25)$$

where P_{pulse} be the pulsation period.

There are mainly two systems have been well observed long enough (yr) to allow detection of the $- P_{\text{pulse}} / P_{\text{pulse}}$ appropriate to the neutron star case. Of course, any accreting star that is spun up for a sufficiently long time will enter the fast roation regime $w^* > 1$. Hence at the equilibrium pulse period P_{eq} we have $R \ll R_M$ and

$$P_{\text{eq}} = 3M_1^{-2/7} R_6^{-3/7} L_{37}^{-3/7} u_{30}^{6/7} \text{ s} \dots\dots\dots (26)$$

Since $u_{30} \leq 1$ for neutron stars, it is obvious that only a few of the X-ray binary acceptors may be near their equilibrium periods. In principle, one would like to know the ‘spin history’ of all types of magnetized accreting stars and relate this to the observed pulse period distribution and spinup and spin down rates.

Concluding Remarks

An acute problem is posed by the large fraction of known pulsating X-ray sources with long pulse periods $\geq 10^2$ s but to their lifetimes 10^3-10^6 years as bright X-ray sources expected evolutionary arguments. One may expect the formation of a supernova with a very short spin period of a fraction of a second. At such spin rates $w^* \gg 1$ i.e. $R \ll R_M$ and accretion seems to be impossible because any matter approaching the neutron star will be thrown off by the centrifugal barrier at R_M . Though it is very easy to describe but hard to evaluate its efficacy due to coupling of matter and magnetic field. The dynamical effects of the stellar magnetic field gives an independent idea of the emission of cyclotron radiation by electrons gyrating around the field-lines, as a result of their thermal motions.

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